What Did Greek Mathematicians Find Beautiful?
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WHAT DID GREEK MATHEMATICIANS FIND BEAUTIFUL?

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1. Types of Questions

The question posed in the title is not one we can readily answer. It overgeneralizes—asking us, in its unrefined state, to lump together Classical, Hellenistic, Roman, and Late Ancient authors, astronomers, and catapult builders, pagans, Christians, philosophers, and pure scientists. My own response concentrates, therefore, on authors of elite, sophisticated mathematical works in the Hellenistic era: Archimedes and his satellites. Nor do the ancients provide us with any help: the proofs were public, and their form became shared, but whatever feelings of aesthetic joy mathematicians experienced concerning them, they kept to themselves, and we need not assume that any two mathematicians had quite the same experience of beauty in reading the same text.

The role of such an inquiry, then, is to suggest the kind of questions we may ask. Negative results are to be expected, and I shall offer mostly those; a very tentative positive thread is pursued throughout the paper. The argument deals primarily not with the kind of explicit views ancients held concerning mathematical beauty,[1] but rather with the kind of mathematical entities that authors, more or less tacitly, found attractive. Whether or not this attraction should even be conceptualized in terms of the "aesthetic" is a difficult question in its own right, belonging more to the abstract theory of the aesthetic than to the concrete history of aesthetic practices. There are two main reasons why I find the category of the aesthetic useful: first, because the discussion will often involve the way in which the mathematical act was *felt*, an embodied experience of words and images, and second, because the practices to be discussed are related to practices of art: literary, visual, and musical.

1. There are two major passages of relevance that I will not be discussing. One is Plato's *Timaeus* 54a–b, where Plato claims that the constitutive triangle of Timaeus' account is the "most beautiful" (κάλλιστον) in some undefined sense. The other is Aristotle's *Metaphysics* M3 (1078a31–b6), where the main point appears to be that one does not need to assume a separate realm of mathematicals, in order to argue that mathematics has to do with good and beauty, particularly because mathematics deals with such terms as order (τάξις), commensurability (proportion?) (συμμετρία), and the definite (ὡρισμένον). Both passages are explicitly tentative: Plato opens up the possibility that other triangles are more beautiful; Aristotle promises to deal with this more clearly elsewhere (which he does not do in his extant writings). The two passages at least prove that it is not totally anachronistic to discuss the beauty of mathematics in an ancient context; they also display the variety of the ways in which such beauty can be discussed, whether as property of mathematical objects (in Plato) or mathematical practice (as it appears to be the case in Aristotle).

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The idea of discussing mathematical beauty is commonplace. It is the animating theme of Hardy 1940, perhaps one of the key reference points to the way in which modern mathematicians understand their discipline. This idea is also at the very fountainhead of the modern tradition of philosophical aesthetics. Francis Hutcheson ([1725] 1969) repeatedly argues that his criterion for beauty—uniformity amidst variety—is valid based on mathematical examples. He apparently assumes that the beauty of mathematical entities is the least controversial. Thus, section 2 of his first treatise starts with the equilateral triangle, which is less beautiful than the square, in turn less beautiful than the regular pentagon, and so on (uniformity is held constant while variety increases); the equilateral triangle, however, is more beautiful than the scalene one (it is more uniform). Section 3 moves on to the beauty of general theorems: the prize example now is *Elements* 1.47 ("Pythagoras' Theorem"), which has infinite variety in it (all those triangles for which it holds true!) and yet is perfectly uniform in all this variety.

This reminds us of a basic aesthetic duality. We naturally tend to think of the aesthetics of the *signified*—the nobility of the tragic hero, the pathos of the poet's love—but we must always force ourselves to turn from the more manifest signified to the self-effacing *sign* itself. The tragedy is so effective not because its hero is noble, but because its plot structure is so tight; the poem is so captivating not because of its pathos of feeling but because of its sequence of sounds. And the same is true in any sign-and-signified pair: it is in the nature of communication that our attention fixes upon the thing signified, but it is also in the nature of experience that our sense of beauty is shaped by the thing making the sign itself and not just by the thing signified. There must always be an aesthetic dimension to the sign. And indeed it is clear that a mathematical text does not produce a sense of beauty by referring to a nice triangle. One can refer to a nice triangle in a boring, pedestrian, ugly way, or one can refer to it in a brilliant, captivating way. And so there must be an aesthetic to the telling of a mathematical argument, a textual aesthetics of this textual object that we call mathematics. Therein must lie much of the aesthetics of the field, and what Greek mathematicians found pleasurable was surely, among other things, a certain way of writing and reading about mathematics.

So there are at least two types of mathematical aesthetics. One has to do with the mathematical object, the other with the mathematical text. I shall concentrate in this paper on the mathematical object, primarily because I have already dealt extensively with the aesthetics of mathematical texts. Once again, I refer specifically to the genre and period I concentrate on here—that is, I have written on the aesthetics of highly literate, elite works especially in the geometrical tradition, written in the high Hellenistic period of Greek mathematics, roughly from the mid-third to the mid-second century B.C.E. (the mathematics of Archimedes and his satellites). In my 2009 book, I argued that in such mathematical works one can discern an aesthetic sensibility directly comparable to that of elite Hellenistic literature (the poetry of Callimachus and his satellites, if you will), concerned with surprise, variety, and the breaking of generic boundaries. One may consider, for instance, the way in which Archimedes builds suspense and surprise by postponing the actual spiral until the middle of *Spiral Lines*, or the actual sphere until the middle of *On the Sphere and Cylinder*; the central role of mixed sciences in Hellenistic mathematics (e.g., in mechanics or astronomy); or the obvious interest of Hellenistic mathematicians in introducing a sense of *poikilia* (variation) through producing a multitude of results of different kinds.

Consider the very first words of the introduction to Archimedes' *Stomachion*:[2]

> As the so-called *Stomachion* has a variegated (*poikilê*) *theoria* of the transposition of the figures from which it is set up . . .

Or the way Apollonius presents Book 3 of *Conics*:[3]

> Book III contains many theorems that stretch belief (*paradoxa*)—useful both for the solution of solid loci as well as for the ⟨finding of⟩ limits of possibility ⟨on problems⟩, of which most—and the most beautiful (*kallista*)—are new . . .

The bulk of the *Stomachion* is lost, but apparently Archimedes' point was that the study essentially involved a hybrid of geometrical and calculatory techniques. From the extant *Conics* III, it is clear that Apollonius aimed at maximal surprise, a sense of maximal medley: results do not anticipate each other in an obvious way, but instead lead in an ever-surprising direction. By being maximally surprising or paradoxical, the results apparently are also maximally "beautiful," and I chose this quotation primarily because it is the unique relevant use of the word "beautiful" by a Greek mathematician. Again, since it is common to understand Hellenistic literature through the lens of a very similar aesthetic, one that prizes surprise, variety, and the hybridization of genre,[4] it becomes natural to suggest that a similar reading of the aesthetics of Hellenistic mathematics may well capture something genuine about the kind of aesthetic pleasure that mathematical authors and readers took in such works. This of course is an argument in literary interpretation, and is subject to the uncertainties of such arguments; it is, however, as useful an argument as we can make in the aesthetic interpretation of a culture. So here is one type of beauty certain mathematical authors might have prized: the beauty of presenting a mathematical work, in literary terms, through an aesthetic we may perhaps call Alexandrian.

This has to do, of course, with the aesthetic of the sign, of the *telling* of the mathematics. The same aesthetic seems to hold, in this case, for the tangram shapes of the *Stomachion* as for the parabolas and hyperbolas of Apollonius' *Conics*. But what is it that Greek mathematicians prized about certain objects themselves? What was the aesthetic of the mathematical object? I repeated above a positive suggestion I made in my 2009 book, concerning the aesthetics of the text. I now wish to follow this with two negative observations

2. Heiberg 1913, 416.2–3. A somewhat improved transcription of Archimedes' *Stomachion* is offered in Netz et al. 2004, but Heiberg's text of the words quoted above is unchanged.

3. Heiberg 1891, 4.10–13.

4. This is indeed commonplace, at least in a tradition culminating in Kroll 1924. Recent scholarship, culminating in Fantuzzi and Hunter 2004, tends to qualify such commonplaces but does not deny them.

concerning the aesthetics of the object. Through this discussion I will tentatively try to construct a positive, alternative account.

2. VISUAL BEAUTY?

Did Greek mathematicians find their object of study pleasing to the eye? We usually do, and so did Hutcheson: the impression I have from reading the second chapter of his Treatise 1 is that he considered the square more beautiful than the regular triangle as a matter of visual perception (and not as some kind of abstract conception; this indeed becomes clear from his setting out of his overall project in chapter 1). Now, I find it hard to tell what my own intuition is concerning the relative beauty of squares and triangles. But I find nothing counterintuitive at all about the idea that geometrical configurations, as such, might possess beauty. Many of us probably share the sense that certain configurations of rectangles are among the most perfect works of art ever produced (I think of course of Mondrian), and, in more general terms, it comes to us most naturally to think of the aesthetics of painting, sculpture, and of course architecture in terms of an underlying, purely geometrical structure. So, did the Greeks do the same? Was there a little bit of the Mondrian in their Euclid?

There is no way of getting into the Greek mathematicians' minds in this regard, but there is a way of getting into their eyes: we can try to reconstruct what they saw, in a certain context at least, when contemplating geometrical figures. I refer to our evidence for Greek geometrical diagrams. This extant evidence is admittedly very small, but it is suggestive and coherent, and it can be supplemented from the evidence we have from transmitted diagrams on medieval manuscripts, which I believe ultimately go back to ancient sources.

We have a few papyri fragments by Euclid, such as *Oxyrhynchus Papyri* (*P. Oxy.*) I.29, or *Berlin Papyri* (*P. Berol.*) 17469, or *Michigan Papyri* (*P. Mich.*) iii 143 (definitions, no figure). These are apparently private copies with extracts from Euclid,[5] most probably in use by (as most recently stated for *P. Mich.* iii 143 [Fowler 1999, 215]) "a grammaticus, making sure he has the text which he wanted to dictate to his students or on which he intended to base his next lesson." The famous Euclid-like ostraca were obviously also private, though perhaps they were a scholar's notes.[6] The only professional,[7]

5. Most significantly, *P. Oxy.* I.29 and *P. Berol.* 17469 do not have letters referring to the figure: apparently they are mere collections of general enunciations.

6. Mau and Müller 1962. A set of six ostraca contain materials relating to Euclid's construction of regular solids in *Elements* XIII. The ostraca were found in Elephantine, at the southern border of the Ptolemaic kingdom. They are dated, on paleographic grounds, to the second half of the third century B.C.E. None of the ostraca survives in its original size but the extant text in the best-preserved ostracon (no. 1) is comparable, perhaps, with a papyrus column or two. The texts offer a set of fully fledged proofs. The handwriting is neat but (as is, after all, to be expected) not that of a student. All in all, this is the equivalent of a mini-treatise on ostraca. There are hardly any parallels to this kind of artifact. Someone in the deep Ptolemaic south was studying, alone or with a teacher, the apex—the Platonic solids—of Euclid's *Elements*. (Perhaps Eratosthenes in his sojourn to measure a solstice in Syene?) At any rate, it is only natural that the relatively well-preserved figure in ostracon 3 (P. 11999, drawn in Mau and Müller 1962, 6) is a freely drawn circle with lines drawn inside it, of a very small size: 2.2 centimeters high for the space of the figure.

7. For the distinction between the private and the professional in ancient papyri, see Johnson 2004.

booklike fragment we possess is *Fayum Papyri* (*P. Fay.*) 9, a second-century C.E. fragment of Euclid's *Elements* I (the end of proposition 39—with the diagram—followed by 41). It may be the unique source we have for a "book" text of literate, elite content in pure geometry. Eric Turner comments (Fowler 1999, 214):

> [The reason why the fragment is of interest] is in the drawing of the figure, and its two surviving letters. It seems to be that these letters were written by the original scribe, but with a different, sharper pen . . . he was taking some trouble . . . perhaps these indications are enough to suggest that we have part of a real manuscript of the *Elements* here.

Perhaps another "book" in the exact sciences to survive from antiquity, with at least a fragment of a geometrical figure, is *P. Oxy.* 4144, a kinematic text argued by Jones (1999, 108–9), very persuasively, to come from an astronomical treatise (perhaps a fairly elementary one?). Other than such fragments, papyri in the exact sciences appear mostly to come from private notebooks, from a less elite tradition of astrological table making, or elementary mathematical problem solving (or even rote learning), with a few exceptions (such as a fragment from a book by Menelaus, possibly, *P. Oxy.* 4133.69–80) where we simply do not have the luck to have a figure preserved.

The two fragments *P. Fay.* 9 and *P. Oxy.* 4144 are therefore what we have to work from. Let us consider the evidence. The first, *P. Fay.* 9, had a clean, but also very lean and monochromatic pattern of sharply delineated lines. Its overall size was less than 4 by 8 centimeters. Specifically, it seems to have taken the width of a column of writing, with the length of perhaps six to eight lines of writing. *P. Oxy.* 4144 was similarly monochromatic. It was much less neat, which may have to do with the fact that it contained a curve. That is, *P. Fay.* 9 did show the use of a ruler (naturally a ruler would be at hand in the making of a genuine "book"), but a compass need not be available to a scribe, and the curve in *P. Oxy.* 4144 was therefore drawn freehand. The figure once again had for it the entire width of the column available (though it seems to use just a small portion on the left-hand side), and the length vacated by the text was a mere 3.5 centimeters (Jones 1999, 108).

The two figures share a few features. They are small. They use the same ink as the main text, and so one color of ink alone. They are purely linear. They occupy the width of the column for a few lines. All of this follows the practices of the writing itself (as is suggested by the observation that they are probably produced by the same scribe): a small, if neat, handwriting, unadorned, monochromatic, repetitive, based on the sharp incision of a penknife.

This, then, is the evidence of two papyri fragments. Should we not just despair of the project of reconstructing the character of ancient diagrams on such a slender basis? Not quite: this miserable evidence is useful as an anchor for our wider evidence. We do after all have all those private, or less elite, papyri fragments; and then again we have the medieval parchment copies. Both types of evidence amplify and support our overall impression. In papyri diagrams, straight lines are typically drawn with some care, but curves are freehand and very impressionistic. The *Ars Eudoxi*, or Louvre *Greek Papyri*

(*Pap. Gr.*) 1, is a very early papyrus perhaps roughly contemporary with Archimedes and Apollonius, and contemporary with the Euclid-like ostraca—an interesting astronomical text that very powerfully shows the character of such freehand circles. Figures typically remain fairly small (the *Ars Eudoxi* has some of the bigger drawings in the papyrus evidence for the ancient exact sciences). The overall sense is of a small, unadorned linear network. The evidence from medieval manuscripts once again confirms the same impression; the deviations from ancient sources are easily explained as the consequences of a new medium and a new context. We may once again look at the figure of *Elements* I.39, this time from the famous Bodleian MS D'Orville. That the figure is essentially the same as *P. Fay.* 9 is evident. In general, medieval manuscripts tend to be bigger (but, then, they derive from big parchment folia). Circles are drawn with the compass (as are all other curves[8]). Once again, the principle is that one uses whatever tools one has in one's scriptorium, and so, the more lavish institution of the Middle Ages (which had experience not just in calligraphy but also in the drawing of miniatures) had a more lavish set of instruments. For a similar reason, medieval diagrams are often bichromatic, with perhaps the labels introduced in red (the labels are therefore a kind of "rubric"). All in all, medieval diagrams are more "distinguished," as in general the medieval parchment was more distinguished than the ancient papyrus. But the overall pattern of configuration encoded within the diagram, even in the medieval manuscripts, seems to be schematic and flat, always keeping to the principle of a simple, unadorned linear network. I believe this pattern of configuration derives from the ancient sources themselves, as argued in Netz 2007 and Saito 2008.

Let us compare this first with a certain modern rendering of a Greek mathematical content, then with an independent mathematical tradition. We may set side by side Archimedes' diagrams in two nearly opposite traditions. One is (what I believe to have been, based on manuscript evidence) the ancient drawing. The second is the reworking of ancient science in independent visual traditions of the scientific revolution: perhaps the most striking example for Archimedes would be Rivault's edition of 1615.

One thing one would notice immediately, comparing the ancient tradition with that of the scientific revolution, is that editors like Rivault provide pretty pictures to look at, of a pretty object. One often comes across lavishly represented three-dimensional objects, with charming effects of light and shade. The ancient diagram, in contrast, is not pretty, primarily because it is not a picture and in a way it is not of a thing. What is typical of ancient mathematical diagrams is that they deliberately avoid the visual evocation of a spatial object. Instead, they are schematic, almost conceptual maps of the topological pattern of overlap and intersections within a configuration. There is an excellent epistemological reason why this should be the case,

8. Toomer 1990, lxxxv: "as seems to be the universal rule in medieval mss., conics are drawn with compasses and appear as combinations of regular arcs." It is conceivable that ancient conics were drawn more "accurately," as they, as well as circles, would be drawn freehand.

which once again confirms our sense that we are indeed recovering the ancient form of ancient diagrams. If you wish to rely on the diagram as part of your geometrical reasoning, you should rely on its topological properties rather than its metrical ones. (I expanded on this argument in Netz 1999, 33–35). There are obvious reasons why a metrical diagram is unreliable. Basically, any act of visualization is metrically inexact; there are other reasons, studied in depth by recent logicians,[9] why topological diagrams are reliable. Put very crudely, it is hard to draw a diagram such that it is topologically false. Now, Greek mathematicians, unlike their modern counterparts, did rely on the diagram as part of their reasoning, and for this reason made a conscious decision to avoid a pictorial diagram, relying instead on fairly schematic representations. The ancient diagram is very pragmatic, in this regard: it is a logical tool. Hence the visual impression of the ancient diagram—a hasty sketch, almost a thumbnail. It is perhaps best comparable to a drawing on the small blackboard one finds in a Mathematics department's lounge, serving roughly the same function (Greek mathematicians communicated across the folds of a papyrus roll).

We see a fundamental fact. Greek mathematics foregrounds persuasion: it is all about the ability to prove something in an incontrovertible way. For this reason, everything is organized around the essentially textual phenomenon of the argument. The key way of understanding the aesthetic of this type of mathematics is for this reason textual—hence, perhaps, the relative ease with which one can find a "Callimachean" aesthetic in Archimedes.

This is not necessarily the typical road for a mathematics to take. Most cultural forms of mathematics, developed independently of Greek influence, do not foreground persuasion so much. Indeed, very often the argument is elided and just the bare results are reported, perhaps with some indications of an underlying calculation. Consider, for instance, an interesting cultural form. In the Edo period, Japanese scholars would dedicate groups of mathematical tablets at Buddhist or Shinto temples. The format of a tablet was fixed: a very brief statement of the result obtained, headed by a richly illustrated figure of the geometrical configuration. To watch such a group of tablets (the reader is strongly encouraged to consult the gorgeously produced volume by Fukagawa and Rothman[10]) is to be immersed immediately in the enormously visual culture of Japanese temple aesthetic. The overriding sense of beauty is apparent in the calligraphy as it is in the figure, which is carefully executed with spectacular color. The entire set of problems is planned as a unified visual whole, down to such details as the grain of the pieces of wood. The set is also huge: 162 by 88 centimeters for the entire set (which is the

9. See Shin 2008 and references there.

10. Fukagawa and Rothman 2008. See, e.g., plate 5; the book surveys many tablets of this kind—the practice was very widespread—and it should be noted that supreme visual aesthetic did not mean any slackening of the purely intellectual content, which is very difficult and sophisticated. Unfortunately, Fukagawa and Rothman systematically overwrite the original texts with modern algebraical transcriptions, so that a nonreader of Japanese cannot judge the more purely textual aesthetic—but the impression, just based on the contents of the problems, is that the goal was somehow to "entrance" the viewer, catch him or her off guard, send him or her on a long chain of thought. An object of reverie, perhaps.

level at which this should be considered, as this is the level to greet the eye). This is not what Greek mathematics was like; one thing we can say for certain is that Greek mathematicians did not have a primarily visual aesthetic. This is perhaps surprising but I think also a very clear result.

So let us pause to consider the evidence. In some other contexts (in the modern reception of Greek mathematics, or in non-Western mathematical traditions), the beauty of the represented geometrical object could, at least sometimes, be directly celebrated in a visual way. It is also a standard philosophical position (and an intuitive one) that geometrical configurations may at least sometimes be beautiful, and it is hard to resist the intuition that many of the geometrical configurations studied by Greek mathematicians were indeed beautiful (I usually think of the spiral in this regard).

I do not think that our argument should be that the Greeks, unlike us, did not perceive beauty in geometrical objects. There is, after all, the direct evidence of the *Timaeus* as well as, ultimately, the preponderance of likelihood that such aesthetic intuitions as we share need not be entirely cultural in origin. Rather, what we seem to notice is a certain choice: to make the visual representation subservient to textual encoding, even in the context of the supreme visual object of the geometrical configuration. This, in and of itself, suggests a certain aesthetic sensibility: that the visual acquires its significance through its participation in a textual practice, through its narrative service. The visual representation itself may be seen essentially as a textual act calling for the kind of decoding we usually expect from narrative: the aesthetic experience resides not so much in the taking in of a marvelous visual object, but rather in the pleasurable act of interpretation, taking a mere linear network and "reading off" its intended spatial significance. This, in turn, is not so far away from the aesthetic of narrative suspense described in Netz 2009. But to create my own narrative suspense, I set this thread aside, to be picked up at the end of the next section.

3. MUSICAL BEAUTY?

The notion that mathematics has something to do with the realm of the beautiful rings of course as "Pythagorean," and the natural way for an ancient author to think of an aesthetic interface of mathematics would be with music: after all, Pythagoreanism may have made its greatest impact through a mathematical theory of the beauty of sounds.[11] This operated through an identification of musical harmonies with numerical ratios.

How to study whether Greek mathematicians cared for musical ratios or not? We need to narrow our field. So let us first of all distinguish between problems and theorems. Perhaps it may be argued that problems are valued primarily for their *manner* of obtaining their result, while theorems are valued,

11. The evidence is best studied in Huffman 1993, 364–80, and 2005, 402–80. It appears that for authors such as Philolaus and Archytas, music was the key example justifying the prominent role of mathematics in a teleological system. This worldview arguably formed an essential background to Platonism, as to its ancient critiques. In this sense, the ancient value of mathematics was primarily related to the mathematization of music.

at least in part, for the result itself.[12] If so, it is useful to ask primarily about the content of theorems. Now, once again, some theorems are there for the sake of other theorems; they are links in the deductive chain (Mueller 1981 is a classic study in the interpretation of such chains). Other theorems appear to serve as more explicit goals in and of themselves. It is useful to concentrate, then, on such goal-theorems.

What do such Greek goal-theorems typically prove? While some results are purely configurational (a line drawn in a certain fashion will cut another line; there are several results of this kind, especially in Apollonius' *Conics*), most results are indeed metrical. Among metrical results, most take the form of equalities and proportions.

The very earliest attested results from Greek mathematics are of equalities: Hipparchus of Chios showed for four different lunule combinations that they are equal to certain simpler objects.[13] The period is Classical, but the aesthetic is obvious and will be apparent in the Hellenistic period as well: a fascination with an unsuspected equivalence between what are apparently radically distinct objects, the curved and the straight.

Many other results are of proportions. The best example is actually a tool, but it is subjectively clear that this result was considered interesting in its own right.[14] This is the set of basic results for conic sections. You may say that this is why conic sections are so spectacular: they give rise to unsuspected proportions. For example, a parabola gives rise to a proportion between squares and lines: as a certain square is defined by the conic section to another one, so are two lines to each other. The result is extremely useful in that it allows us to produce a kind of dimensional conversion (turning proportions between plane figures into proportions between line segments), and for this reason conic sections are applied everywhere in advanced Greek geometry. But it is also spectacular and surprising in its own right, juxtaposing the one-dimensional with the two-dimensional. The results are now preserved in the first book of Apollonius' *Conics*, but they were proved earlier, perhaps even in the fourth century. They certainly form one of the key moments in the history of Greek mathematics.[15]

In both cases—quadrature of lunules, or the elementary theory of conic sections—it is significant that the result is not visible. You do not see the equality between the lunule and the triangle: the entire point is that something

12. I did try to argue for this claim in Netz 2000. It is of course ultimately no more than a guess concerning ancient values for which no direct evidence remains.

13. These results are still best read in Bulmer-Thomas 1939, 234–53. The text is extracted from Simplicius' commentary to Aristotle's *Physics* 1.2, and one awaits its eventual translation in the Ancient Commentators Project.

14. A major trend in the growth of mathematics is, paradoxically, the transformation of original tools into ultimate goals (one studies groups so as to classify equations, but ultimately the study of groups is found to be gripping on its own account). It appears that conic sections are the prime example of this kind of trajectory in Greek mathematics.

15. The best guide to Apollonius' *Conics* is Fried and Unguru 2001, arguing for the geometrical significance of the treatise: thus, it is not purely a tool, engaged with allowing certain transformations on equations and proportions, but is more directly a study in the characteristics of certain striking curves.

that was there, hidden in plain sight, suddenly reveals itself through the force of pure thought. As for conic sections, here the relation is invisible as a matter of principle, because it involves not one but infinitely many relations simultaneously. The relation between the two segments can effectively be any relation in the world: they vary infinitely. But as they vary, they also covary with the squares they define. A true proportion statement is really a claim about an infinitely extended covariation, which is therefore embedded in the visible diagram but in principle transcends it. Once again we note the sense that one needs to decode a representation, to immerse it within a certain textual meaning, in order to gain its hidden sense.

Leaving this aside, we may simply note at this point that neither equalities nor extended covariations are the kinds of ratios and proportions that arise in music and that intrigued Pythagoreans. Rather, what Pythagoreans seem to value most is a proportion involving a constant value, not a covariation: A is to B as a certain constant ratio, that of (the given) C to D. C and D could in principle be just any line segments. In a sense, this is what Archimedes does in the first proposition of the *Measurement of the Circle*, where he shows that a circumference is to a diameter as a certain line segment in a certain triangle is to another line segment. In principle, one could have interesting results that reduce various relations into fixed line segments, the ratio between fixed line segments filling the role of numbers. In practice, this does not happen: when Greek mathematicians are interested in constant ratios, they wish to give such ratios a numerical name. And so Archimedes is not satisfied by the first proposition and proceeds instead, famously, in a much more complex later proposition, to provide a boundary value for this ratio. The ratio in question—which we now call π—is found to be smaller than three and one-seventh but greater than three and ten seventy-firsts. Indeed, I have argued in a previous article (Netz 2003a), dealing with Archimedes' *Arenarius*, that a major motivation for a Greek mathematician could be to verbalize a mathematical object: to provide it with a Greek name.

This may be part of the fascination of a musical ratio. What Pythagoreans valued above all are ratios of particular kinds, namely multiples ("double," "triple," etc.) and epimorics (such as "one-and-half-again," "one-and-a-third-again"). Their shared property is that they are encoded by words in natural Greek: *diplasios*, *triplasios*, *hêmiolios*, *epitritos*, and so on. The fundamental claim of Greek mathematical music theory (most skillfully presented in Ptolemy's *Harmonics*, as discussed by Barker in this issue), is that all musical harmony is explicable in terms of multiples and epimorics, in particular those limited to the basic numbers 1–4.[16]

16. The last constraint may once again be rooted in natural Greek: it appears that while *hêmiolios* and *epitritos* are real Greek "words," more complex forms from *epitetartos* upwards have only a technical arithmetical meaning. For whatever it is worth, a TLG search yields 1907 occurrences of ημιολι, 1875 occurences of επιτριτ, 299 of επιτετρατ, and 63 of επιπεντ. A few of the επιτριτ occurrences are from sources that are not strictly arithmetical (e.g., philosophical, including authors such as Xenophon, Theophrastus, and Philo Judaeus). Note also that this term also has a separate grammatical meaning (and is sometimes cited by grammarians), but επιτεταρτ is used by only six pre-Byzantine authors: Nicomachus,

The question, then, can become fairly narrow: how often do multiple and epimoric ratios, in particular those limited to the numbers 1–4, appear in the goal-theorems of Greek mathematicians? Let us consider the evidence from Apollonius, Euclid, and Archimedes.

Apollonius may be quickly dismissed. The extant *Conics*, at least, shows no interest in numerical ratios (with some interest in mere configuration in Books I, IV, and VI). The basic results have to do with surprising covariations and equalities (especially Books II, III, and VII). Book V, perhaps the most spectacular of all, deals with conditions under which certain lines are the smallest of a certain class, a result hard to classify in any simple quantitative terms. In short, Apollonius' interest in conic sections clearly is not musically motivated.

Euclid's *Elements* is mostly a collection of instrumental results. Some of the results are there because they are widely applied throughout Greek mathematics; a few results are no more than interim steps required to derive those more widely usable results. Thus there should be less evidence for the ultimate aesthetic sensibilities of Greek mathematicians in this book as a whole. But it is clear that Book XIII has a different character. Being the final book, it functions perhaps as Euclid's example of what real mathematics looks like, and the same might be true, I think, for Book XII. These are both books in solid geometry, a field in which Euclid has only one truly elementary book (Book XI, extending the plane geometry of Book I). It is perhaps relevant that both Books XII and XIII are possibly related to major mathematicians (XII to Eudoxus,[17] XIII to Theaetetus[18]). Of course, Proclus also saw, specifically, a Platonic significance in Book XIII;[19] he could perhaps have been justified in doing so.

Book XII is fundamentally about two results: that a pyramid is one-third of its encasing parallelepiped, and that, as a consequence, a cone is one-third of its encasing cylinder.

Book XIII is at its surface about constructing regular solids, but most of its mathematical action is about a series of fixed ratios. Some of the results are preliminary fixed ratios for polygons, clearly lovely on their own but fundamentally employed as tools; the point is to find the ratios for sides of regular solids and the radius of the sphere in which they are inscribed.

Ptolemy, Iamblichus, Porphyry, Theon of Smyrna, and Hippolytus (the word thus has no B.C.E. citations). LSJ cites economic and legal usages of *hêmiolios* and *epitritos*, but not of the higher epimoric ratios. I suspect that to the Greek ear, at least up to the Hellenistic world, *hêmiolios* and *epitritos* would sound like genuine words, but *epitetartos* like a mere arithmetical neologism.

It would be a momentous achievement in musical theory when the tyranny of the *tetraktys* was finally removed and ratios involving the numbers 5 and 6 admitted into musical theory, making possible a recognition of the third—and not just the fifth and the fourth—as a significant musical relation (Taruskin 2005, 1: 587: "But, said Zarlino, there is nothing special about the number four, and no reason why it should be taken as a limit. Ah, but six!").

17. That is, Archimedes twice (in the introductions to *On the Sphere and Cylinder* and the *Method*) ascribes the main results of Book XII to Eudoxus.

18. The discussion in Heath 1921, 1: 158–62, may still serve as a useful starting point: several late sources, none decisive, but taken together of some weight, suggest that Theaetetus made a significant contribution to the study of the regular solids.

19. Proclus *In Eucl*. 68.20–23.

In some cases no numerical ratios can be found, and then Book X kicks in (this, in fact, is the only application of Book X in Greek mathematics, and its apparent raison d'être). This, in turn, is related to the point made above, that Greek mathematicians would not take the ratio between a couple of fixed linear segments as the equivalent of a real number; instead, they seek actual verbalized number terms. In this case, since no verbalized number term is available for those irrational numbers, Euclid picks instead another form of verbalization, which is not quantitative but is instead classificatory: the number is irrational of a certain *kind*. (*Elements* X is engaged in the classification of irrational numbers, and giving names to such classes.)

The goal, then, of the self-sufficient books of Euclid's *Elements* is to show certain numerical ratios and, more generally, to verbalize certain ratios.

Let us now recall some of the major results obtained by Archimedes. He shows that a sphere is one-and-a-half the cone contained by the cylinder that contains it; that its surface is four times its greatest circle; that a spiral line encloses an area one-third of its enclosing circle; and so on and so on. There are so many results in the works of Archimedes that take the form of a proportion with a fixed numerical ratio that we should better turn, at this point, to a list summing up all those results from Archimedes, as well as Euclid:

Numerical Ratios: Data from Euclid and Archimedes

(For ease of reference, I quote all ratios in modern numerical notation; almost all are in strict verbal form in the original.)

Major Results of Self-Sufficient Books of Euclid's *Elements*

XII.7 cor	1:3	Prism a third of a parallelepiped
XII.10	1:3	Cone a third of a cylinder

An Entire String of Results in *Elements* XIII

1	5:1 (in square)	Line segments in proportion ("golden section")
3	5:1 (in square)	Ditto
4	3:1	Squares defined by "golden section"
12	3:1 (in square)	Inscribed equilateral triangle
13	1.5:1 (in square)	Inscribed pyramid's side
14	2:1 (in square)	Inscribed octahedron's side
15	3:1 (in square)	Inscribed cube's side
16 cor.	5:1 (in square)	Inscribed icosahedron's side

Major Results by Archimedes in Numerical Ratio Form

On the Sphere and Cylinder I

33	4:1	Sphere's surface to great circle
34	4:1	Sphere to inscribed cone

Measurement of the Circle

3 $3 + 1:7 >$ Circle's circumference to diameter $> 3 + 10:71$

On Conoids and Spheroids

22	3:2	Paraboloid to inscribed cone
28	2:1	Hemispheroid to inscribed cone

Spiral Lines

19	2:1	Spiral linear ratio
24	1:3	Spiral area ratio (major result of the treatise)
25	7:12	Spiral area ratio extended
27	2:1, 3:1 . . .	(extendable ad infinitum); 1:6

Planes in Equilibrium II

8	1.5:1	Axis of parabolic segment cut by center of gravity

Arenarius

"Thousand myriads of the Seventh Numbers"
(Quasi-ratio: upper boundary on number of poppy seeds to fill the universe, i.e., volume ratio between universe and poppy seed. "Seventh numbers" refers to a numbering system introduced by Archimedes.)

Quadrature of the Parabola

17, 24 (same result, different route):

$1\frac{1}{3}$:1 parabolic segment to inscribed triangle

Floating Bodies II

The treatise as a whole builds up conditions on stability of a paraboloid immersed in water dependent on ratio of axis to side, with tipping points—literally—at the following ratios:

1.5:1
4:15

Method

Two key original results, for figures inscribed within cubes, being respectively, of the cube:

12–15	1:6
16?	1:2

Further results are recalled and proved through the *Method*:

1	4:3	Parabolic segment to inscribed triangle
2	4:1, 3:2	Sphere to inscribed cone, cylinder to inscribed sphere
3	3:2	Cylinder to inscribed spheroid
4	3:2	Paraboloid to inscribed cone
5	2:1	Axis of paraboloid cut by center of gravity
6	5:3	Axis of hemisphere cut by center of gravity

Clearly Archimedes put great value in these findings about numerical ratios, as indeed we can see from his comment in the introduction to *On the Sphere and Cylinder*, where he explicitly compares his results to those of *Elements* XII, indeed comparing himself to Eudoxus:

> In nature, these properties always held for the figures mentioned above. But these ⟨properties⟩ were unknown to those who have engaged in geometry before us—none of them realizing that there is a common measure to those figures. Therefore I would not hesitate to compare them to the properties investigated by any other geometer, indeed to those that are considered to be by far the best among Eudoxus' investigations concerning solids: that every pyramid is a third part of a prism having the same base as the pyramid and an equal height, and that every cone is a third part of the cylinder having the base the same as the cylinder and an equal height. For even though these properties, too, always held, naturally, for those figures, and even though there were many geometers worthy of mention before Eudoxus, they all did not know it; none perceived it.

Now, the idea that there are certain relations "in nature" that the mathematician brings to our notice is certainly suggestive of a Pythagorean-like interest in an underlying reality expressed in simple, numerical, musical terms. And I think there is some evidence suggesting this is not the case.

First, and obviously, quite a few of the numbers we have on hand involve 5 and 7, which, to Greek music theorists, do not have a musical significance. Those are 5:1 (in square) with the golden section, in *Elements* XIII; and then, from Archimedes, 7 in the number Pi; 7 in the *Arenarius*; 4:15 in one of the tipping points in *Floating Bodies*; 5:3 in finding the center of gravity of a hemisphere.

Of course it could be argued that those fives and sevens come out of the nature of things. Archimedes cannot help it if paraboloids tip that way. This is not quite the whole truth—after all, the point is that Archimedes thought a ratio involving the number 7 is neat and simple enough, in his arbitrarily chosen, simplified boundary value for Pi, as well as in the arbitrarily defined problem of the *Arenarius*. And he and Theaetetus thought that results involving 5 are elegant enough and worth being published. But there is a deeper point still: if anything, we should be aware that the quasi-musical numbers involving 2, 3, and 4 come out *naturally* from the type of objects studied by Archimedes. Archimedes never did hit upon this generalization (Cavalieri did[20]), but most of his numbers are really the same numbers over and over again. Over linear functions, integration gives rise to 1:2, over quadratic functions, to 2:3. A triangle (a linear function) occupies half of the rectangle surrounding it; a parabola (a quadratic function) two-thirds. Combine 2:3 with 2:1 and you get 4:3, the ratio of the parabola to a triangle. Essentially most of the numbers found by Euclid and Archimedes can be arrived at through a combination of this kind. The cone, for instance, may be considered as a

20. For a succinct account, see Struik 1986, 214–19. The interest in this observation is that Cavalieri's methods were directly comparable to those of Archimedes: his generalization represents a different kind of project from that of Archimedes. Archimedes sought striking, individual results: authors of Cavalieri's generation sought methods with which to revise Archimedes.

rotated triangle; the rotation gives rise to a quadratic relation, since circles are to each other as the squares on their radius. Thus the original triangle is half of the rectangle surrounding it, but the rotated triangle (the cone) is one-third of the rotated rectangle (the cylinder). So we may argue that the musical overtones are mere coincidence, an artifact of the rather simple way in which integration—and musical harmonies—both operate, each in its own realm. The ratio 1:2 happens to be both the ratio of the octave, as well as the ratio of a linear function; the ratio 2:3 happens to be both the ratio of the fifth, as well as the ratio of a quadratic equation; and through combinations of these we may further derive 4:3, the ratio of the fourth or that of the parabola to the triangle.

But the main piece of evidence on which I wish to concentrate is the following. It is a single case, but I consider this to be a smoking gun, because it does reveal something crucial about Archimedes' priorities. In the *Method*, Archimedes' main original result (propositions 12–15) concerns a cylindrical shape cut by a slanted plane passing through the middle of the cylinder. This shape is enclosed within a triangular prism, and Archimedes effectively shows that the prism is one-and-a-half the cylindrical shape. This is a good musical epimoric ratio, 3:2, and it is genuinely what Archimedes effectively obtains. It is the number that comes out most naturally from the geometrical setting of the question.

Let us look more carefully at proposition 14. Archimedes first derives this result, fundamentally by showing how this shape is closely related to the parabola. Having derived this result, however, he does not stop at the ratio 3:2 but instead goes on to manipulate his number, through what we may well consider a mathematically unmotivated passage where he mentions that the prism in question is one-fourth of the enclosing cube so that the cylindrical shape is one-sixth the enclosing cube. This, finally, is the result Archimedes flags in the introduction and in the enunciation of the theorem.

In other words, Archimedes had on hand a fine musical ratio, one and a half, and then he made a deliberate choice to present his result as the neat, but musically meaningless value 1:6.[21] Why he should do so is obvious: the one-and-a-half value involved a rather clumsy, ad hoc geometrical figure: it was the ratio of the cylindrical cut to a certain arbitrary triangular prism. The 1:6 value, on the other hand, involves a very simple geometrical figure: the prism enclosing the original cylinder (tellingly, Archimedes concentrates further on the special case of the cube, making the figure maximally simple). So we see, in fact, a balancing act: you wish to have numbers that are as simple as possible, but also you wish to have those numbers relate together figures that are also, themselves, as simple as possible. Most important, there is no ideological demand to stick with the simpler numbers rather than the simpler objects. The balancing act is decided pragmatically.

21. Besides going beyond the numbers 1–4, the ratio 1:6 is strictly nonmusical in that its "meaning" (two octaves and a fifth) goes beyond the space of two octaves within which all of Greek music was conceived. This is an "ultraviolet" kind of ratio.

We may consider similarly a few other nonmusical ratios. Why should Euclid say the cone is one-third the cylinder? (What is one-third? An octave and a fifth? Who ever was interested in this harmony?) Could he not say instead that the cone divides the cylinder into two parts, one twice the other? (An octave!) Or Archimedes: why did he need to find the center of gravity of the hemisphere at three-fifths the radius? Could he not find a point equidistant from the surface as the center of the weight is distant from the center of the sphere, and then say the distance from the center is hemiolic (a fifth!) of the gap between the two points? The general point is that one is practically never forced *not* to express integer ratios in the musical vocabulary of one, two, three, and four. All it takes is some ingenuity in defining the objects between which the ratios are determined. The example of the *Method* was striking, in that Archimedes had to show ingenuity so as to define the objects (and so the ratios) *away* from the purely musical vocabulary of one, two, three, and four. But the general rule is that nowhere do we see any effort to define ratios so as to make sure the musical vocabulary holds. In short, we see no interest in limiting oneself to the numbers one, two, three, and four, and we conclude that Greek geometry does not reveal any musical, "Pythagorean" aesthetic. Proportions are crucial, but they are prized for reasons other than the musical: they are prized for their simplicity. Of course, such proportions often coincide with musical relations. But this is because such musical relations, independently, were prized by musical authors for a similar simplicity (which may reside even in the verbal simplicity of a word in natural Greek). That Archimedes comes up with many numbers that a musician could have appreciated is therefore a coincidence, showing not the musical influence on geometry but the shared aesthetic of the various branches of Greek mathematics.

4. CONCLUSION

The imaginary exercise pursued above—trying to look for alternative ways Archimedes *could* have packaged ratios—is revealing. It reminds us that mathematics involves choice. Mathematical facts are unalterable, but they can be presented in myriad ways. The fact that the center of gravity is just at this point does not yet force Archimedes to describe that point in the terms of three-to-five. Such choices are ultimately underdetermined by the mathematical facts and it for this reason that we are justified in looking for their aesthetic motivations. And we seem to have learned something about Archimedes' sensibilities: he seeks primarily the surprising closure of a simple ending to a narrative quest. You start out seeking a complex object, a cylindrical cut; you end up with a simple word, "six-times," linking up the cylindrical cut with another simple object, a "cube."

Let us look even more closely at the surface texture of this result in Archimedes' *Method* 14. The figure reproduced here as Figure 1 is the (unique) manuscript evidence we have for the diagram: an image from the Archimedes Palimpsest. It is a figure drawn by a Byzantine scribe around the year 975 C.E., but since the codex as a whole appears to be very

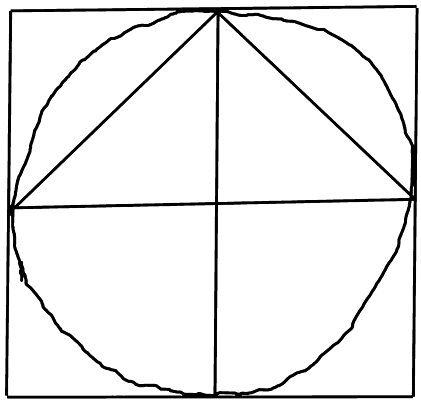

FIG. 1. Diagram of proposition 14 of Archimedes' *Method*. Image from the Archimedes Palimpsest, reproduced by the author, using Photoshop and freehand tracing.

conservative in its diagrams, I assume this is not far from a Late Ancient source and so, conceivably, from the original drawing. The version reproduced here is my own Photoshop tracing, for which I used a line tool to reproduce straight lines but traced the circle freehand—as, indeed, the original papyrus scribes are likely to have done. Notice how the original figure is marked by an extraordinary schematic approach, which once again might be taken as a sign of authorial status (would a later scholastic reader not attempt to make the diagram more accessible?). We see a square, with a circle and a triangle inside it. The square is at the foot of the cube and the cylindrical cut. The triangle, almost incredibly, is there to represent a parabola. So we see once again the unadorned, schematic linear network, a mere toplogical set of directions for reconstructing the subject matter of discussion.

Let us also get a sense of the discussion itself. I quote here two passages from *Method* 14, following the translation in Netz, Saito, and Tchernetska 2002, 111–13:

> So the plane shall make a right-angled triangle in the prism cut off from the whole prism, of which ⟨=triangle⟩ one of the ⟨sides⟩ around the right angle shall be MN, while the other ⟨shall be⟩ the ⟨line⟩ drawn up from M in the plane on GD, right to the ⟨line⟩ GD, equal to the axis of the cylinder, and the hypoteneuse ⟨shall⟩ be in the cutting plane itself; so it shall also make a right-angled triangle in the segment cut off from the cylinder by the plane that was drawn through EH and ⟨through⟩ the side of the square opposite GD, of which ⟨=triangle⟩ one of the ⟨sides⟩ around the right angle shall be NS, and the other ⟨shall be⟩ in the surface of the cylinder drawn up from S, right to the plane DH, and the hypoteneuse ⟨shall be⟩ in the cutting plane. And the triangles are similar. And since the ⟨rectangle⟩ contained by MN, NL is equal to the ⟨square⟩ on NS (for this is obvious, as has been said), it shall be: as MN to NL, so the ⟨square⟩ on MN to the square on NS. . . .

> . . . there are certain magnitudes equal to each other—the triangles in the prism; and there are other magnitudes, which are lines in the parallelogram DH, being parallel to KZ,

> which are both equal to each other and equal in multitude to the triangles in the prism; and those triangles, in the segment cut off, shall also be equal in multitude to the triangles that come about in the prism, and the lines taken away from the lines drawn parallel to KZ between the section of the right-angled cone and EH, shall be equal in multitude to the ⟨lines⟩ drawn parallel to KZ in the parallelogram DH, it shall be, as well: as all the triangles in the prism to all the triangles taken away in the segment cut off from the cylinder, so all the lines in the parallelogram DH to all the lines between the section of the right-angled cone and the line EH. . . .

These two passages, between them, mark the major movement of the proof. First, a certain relation is established between certain triangles; then the triangles are generalized to produce a result for the entire cylindrical cut. Following the second passage, Archimedes will be able to establish the hemiolic ratio of the triangular prism to the cylindrical cut which, as explained above, he will finally transform into the less musical (but more concrete) ratio of 1:6, to the enclosing cube.

I do not wish to try to explain now the (very complex) logical structure underlying Archimedes' argument. Instead, my point is that the diagram and the text quoted above form a continuous whole: they are both very hard to decode. Archimedes does not help the reader to interpret the figure: A triangle for a parabola? A square for a cube? You have to work this out for yourself! The very complex three-dimensional figure of the cylindrical cut has to be imagined on the basis of a circle in a square. Similarly, Archimedes does not explain why a certain rectangle is equal to a certain square—the key move in the first part—but merely states this result is obvious (referring back to an earlier passage where he makes the same claim). Nor does he explain how the generalization of the individual result is justified in the second part. Archimedes sets up the scaffolds for an argument but then mystifies the reader, leaving him or her to figure out alone the argument's validity.

We seem to find a maximal contrast between narrative (and visual) opacity, on the one hand, and simplicity of result, on the other hand. This contrast, I believe, may have been important for Archimedes' aesthetic. The simple structures revealed by Archimedes are, at the beginning of the deductive chain, invisible. They are hidden in plain sight. Through the course of the proof, we are led through a very difficult texture of writing, to a sudden revelation: what was hidden in plain sight is verbalized in simple form. The schematic diagram and the illegible text are suddenly endowed with clear, crisp meaning.

We are led back to two principles of the aesthetic of the mathematical text: the juxtaposition of maximally different components, and the narrative of surprise. We juxtapose the visible and the invisible; we gradually make the invisible visible. More precisely: we make the invisible *verbalized*. In a slogan, then: the beauty of Greek mathematics—at least, the kind whose prime exponent was Archimedes—resided in *verbalizing the invisibly visible*.

If so, the beauty and the pleasure that mathematicians find is ultimately tied not to a specified metaphysics of their field, but rather, if anything, to an epistemology of their field. When we think about the epistemology of Greek mathematics we tend to think about it as prime example of rigor. But there is something else: Greek mathematics is a prime example of discovery. The mathematician, after all, deals with *heuresis*, with the finding of new results. More than that, he finds that which was hidden in plain sight but which, for sight alone, would have to remain hidden and could be made seen only through the mind's eye. Thus the pleasure of doing mathematics is built on a faith in the ability of humans to find knowledge that goes beyond sense perception alone and to express such knowledge in human language.

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RESPONSE TO NETZ

ALEXANDER LEE

The question put forward by Reviel Netz, "What did Greek mathematicians find beautiful?," carries with it a larger question about motivations. Even though it is difficult to identify other clear motivations—social, institutional, professional, and so on—for Greek mathematicians' activities, there remains the notion that they found something beautiful about what they did and that the resulting pleasure is what spurred them on in their mathematical pursuits.

Such an inquiry is fraught with potential missteps, and the sorts of complications that one encounters are nicely set forth in Wilbur Knorr's treatment of the issue.[1] The problems stem in large part from the use of the indirect tradition, specifically the philosophically influenced treatments of mathematics. Netz avoids such issues by restricting his use of evidence to the written products of Hellenistic mathematicians and by deliberately leaving out contemporary or near-contemporary commentary by nonmathematicians.[2] In particular, the philosophical treatments by Plato and Aristotle draw us into a metaphysical inquiry, and we are led to think that perhaps there is something beautiful about mathematical objects and the relations that hold between them, and we begin to wonder what these objects are—forms, abstractions, intermediates, or something else—as well as what they are like. But if Netz is correct in locating the beauty of Greek mathematics in the act of discovery, in the recognition of heretofore unseen relations between mathematical objects, then the preoccupations of the philosophers will have taken us quite far from the actual motivations and interests of the mathematicians.

It is thus better methodologically to exclude such external treatments of mathematics from this inquiry. Yet once some conclusions have been reached (even if they are in large part negative and only tentatively positive, as Netz acknowledges), we can make use of them when reexamining the third-party evidence. That is to say, an improved understanding of the aesthetics of ancient mathematics, developed apart from the philosophical evidence, can make some meaningful contribution to our reading of ancient philosophy. This re-

1. Knorr 1986, chap. 1.
2. Netz, p. 426, n. 1 above.